Wise Use Of GeoGebra Supported By An Evaluation Routine

Zoltán Kovács
e-mail: kovacs.zoltan@nye.hu
University of Nyíregyháza,
4400 Nyíregyháza, Hungary

Zsuzsanna Cole
e-mail: zcole@tenafly.k12.nj.us
Tenafly Middle School,
10 Sunset Lane Tenafly, New Jersey, 07670

Abstract

The research question guided this study was: What kind of review related to dynamic geometry worksheets support teachers applying dynamic geometry applications in the classroom. We argue that besides teachers' personal evaluations a robust peer review is needed for this purpose. The study explores how a course in the teachers' training program could help Mathematics teachers in using dynamic geometry systems in the classroom. This course was based on the recognition that with the growing number of online materials teachers should have strong competencies not only to create new materials but to evaluate existing sources. We claim that introducing an evaluation routine in the training helps teachers in creating, modifying, and utilizing dynamic geometry activities. Based on TPCK paradigm, we complete the evaluation guideline published by other authors and argue that the teacher should take part in the evaluation process of dynamical geometry worksheets.

1. Introduction

What do we mean when we say, 'wise use of GeoGebra', in contrast with 'good' or 'excellent use'? Our view is that there is no definite 'good' solution to technology integration in the classroom. The growing complexity of technology applications is changing the underlying methods and mathematical landscape. Yesterday's 'good' solution in the classroom has an improved version today, solutions have no stopping rule. Just an example in the context of GeoGebra: the appearance of GeoGebraTube, GeoGebraBook, or GeoGebraGroups brought a new view of GeoGebra activities. Moreover, every educator's teaching situation is always unique, occurring in complex cultural, pedagogical and social contexts. Social context may refer to the social background of pupils, but it is also influenced by national standards as curriculum and language.

The research question guided this study was: What kind of review related to dynamic geometry worksheets support teachers applying dynamic geometry applications in the classroom. We claim that the teacher's personal involvement in the evaluation-qualification process is essential in technology integration problems. The first step is the evaluation of the planned activity before the action. We completed the evaluation guideline by [5], and this completed guideline was used in our research.

2. Theoretical underpinning of this research

The main topic of this paper is the end user based intentional evaluation process of GeoGebra worksheets. We include this particular issue in a more general context of technology integration in the learning process. Mishra and Koehler in [11] give the overall framework for our approach. The authors introduce the Technological Pedagogical Content Knowledge (TPCK) as a way of thinking about the knowledge teachers need to understand to integrate technology in the classroom. The system is based on knowledge on technology (T), pedagogy (P), and content (C), and all the possible intersections of these areas (PC, TC, TP, TPC). The authors argue that it is the interactions between and among these components that account for the wide variations seen in educational technology integration. We share the approach that good teaching with technology requires an understanding of the representation of mathematical concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn, and how technology can help reformulate some of the problems that students face.

New technologies are driving necessary and inevitable change throughout the educational landscape. Effective technology use, however, is difficult, because technology introduces a new set of variables to the already complicated task of lesson planning and teaching. The TPCK framework describes how effective teaching with technology is possible by pointing out the free and open interplay between technology, pedagogy, and content. [8]

From a didactical point of view, one of the most problematic issues in dynamic geometry utilization is the relationship between the students' proof activity and use of dynamic geometry worksheets. This phenomenon was widely studied, and Vol. 44 of *Educational Studies in Mathematics* is devoted to this question, see e.g. Gila Hanna's introductory paper [4] and Colette Laborde's review in that volume [9]. Many papers pointed out how interactions in the classroom can develop elements of deductive reasoning ([6], [10]). Dynamic geometry environment enriches students' traditional interactions with the teacher and other students and adds the student-software interaction to the palette. [6] concludes that the dynamic nature of the software influences the form of explanation given by the students. Concerns remain, however, that the opportunity afforded by the software (such as testing analogical figures by dragging and confirming conjectures through measurement) may reduce the need for deductive proof. All the papers in the aforementioned volume refute the idea that proof is endangered by dynamic geometry environments [9].

With a growing number of freely available online GeoGebra worksheets, several authors stress the importance of evaluating online educational materials. Among the first, Hohenwarter and Preiner drew up a system of quality criteria for dynamic mathematics worksheets. In their work [5] the authors constructed a guideline for designing dynamic worksheets and evaluating interactive online materials. Their system concerns four general principles based on research on e-learning: multimedia principle, contiguity principle, coherence principle and personalization principle [3] are highly influenced by cognitive load theory [15]. The key phrases of these principles are the following [5]:

Multimedia principle: Use words and graphics rather than words alone.

Contiguity principle: Place corresponding words and graphics near each other.

Coherence principle: Adding interesting material (i.e. entertaining stories, background music, detailed textual descriptions) can hurt learning.

Personalization principle: Use conversational style. In conversional style the reader is addressed directly.

From the general principles, they derive 16 quality identifiers in three groups: layout of dynamic worksheets, dynamic figures, and explanations and tasks (Table 1).

Table 1: Quality related key words and phrases in [5]

Layout	Dynamic figures	Explanations and tasks
Avoid scrolling	Interactivity	Short, clear, personal style
Short explanation	Easy-to-use	Small number of questions
Few tasks	Size matters	Use specific questions
Avoid distractions	Use dynamic text, it should be placed close to the corresponding object	Refer to applet
	Avoid static text: too much text can clutter the applet	Your audience are learners
	First appearance (i.e. when dynamic worksheet is opened it should be readable)	Demonstration figures (no tasks or questions)

In the spirit of TPCK paradigm, in our guideline we completed Hohenwarter-Preiner's guideline focused on mathematical and pedagogical quality.

The quality assessment of dynamic geometry resources was one of the highlighted aims of the Intergeo project reported in [16]. The Intergeo scheme contains nine classes of indicators concerning the resource quality, namely: metadata, technical aspect, mathematical dimension of the content, instrumental dimension of the content, potentialities of dynamic geometry, didactical implementation, pedagogical implementation, integration of the resource into a teaching sequence, usage reports. The concept of user's review and peer's review also appeared in the Intergeo approach. Though we did not take the Intergeo indicators, we share their point that the quality of the resource depends on its adequacy to the context in which it will be used. Thus, clarifying the educational goal and the school context of the resources is essential, and we demanded that students should identify these possible contexts in our project. (Table 2 summarizes the basic principles of various models.)

In [7] the author focuses on the conceptual design of a new Review System for dynamic materials on GeoGebraTube. The author outlines a paradigm based on direct intentional review by users and indirect automatic review. Intentional review covers the number of "likes", "favourites", comments, and star rating. Possible automatic quality criteria include the frequency of views, the number of materials provided by a user, receiving favourites compared to the number of views, communication with other user, and the frequency of views on a user's profile page [14]. In our view the aim of this evaluation system is the preliminary orientation of a possible user, but it does not replace the teacher's own evaluation work.

Table 2: Quality identifiers based on various models

E-learning based principles [5]	Intergeo's indicators [16]	TPCK backround
Multimedia principle	Metadata	Mathematical viewpoint
Contiguity principle	Technical aspect	Pedagogical viewpoint
Coherence principle	Mathematical dimension of the content	Layout
Personalization principle	Instrumental dimension of the content	Role of dynamic elements
	Potentialities of dynamic geometry	Text and instruction
	Didactical implementation	
	Pedagogical implementation	
	Integration of the resource into a teaching sequence	
	Usage reports	

3. Action plan

The first author delivered a compulsory course for practicing teachers and teacher trainees on technology integration in mathematical education for eight years at the University of Nyíregyháza, Hungary. This paper is based on the findings from the past eight years starting with 2011. From the beginning, evaluation of dynamic geometry worksheets was part of the syllabus. For the evaluation process we drew up an evaluation guideline with points of view specified below, i.e.

1. Mathematical viewpoint
2. Pedagogical viewpoint
3. Layout
4. Dynamical elements
5. Text and instruction.

In our evaluation guideline we kept Hohenwerter-Preiner's identifyers (Table 1). Additionally, we completed the *explanations and tasks* group with the linguistic-grammatical analysis of the text appearing on the screen and introduced two new groups, the mathematical viewpoint and the pedagogical viewpoint. Therefore, the final guideline contains five main groups: mathematical viewpoint, pedagogical viewpoint, layout, dynamic elements, and instruction and text.

*Mathematical viewpoint (*Table 3*).* This viewpoint includes investigation of consequent notations, proper usage of mathematical language, dealing with special cases and exceptions, and mathematical errors.

Table 3: TPCK analysis of the mathematical viewpoint

Identifier	Analysis (TPCK)
Mathematical language and notations	Consequent notations meet mathematical traditions, national traditions, and are in accordance with other teaching sources and teachers' everyday usage. (PC) Unnecessary or over-elaborated notations (i.e. unnecessary subscripts) may cause cognitive overload. (PC) Overriding, moving, deleting, or typographical changing of the notations offered by the software in accordance with the above aspects requires technological knowledge. (T) A basic element of mathematical competences is decoding and interpreting symbolic and formal mathematical language [13], and dynamic worksheets should help the progression of this competence. (TC) We should name objects in worksheets in accordance with other teaching sources. (PC)
Dealing with special cases	Special cases often represent extremal solutions to a problem, and the software should support identifying these cases. (TC)
Dealing with exceptions	Several mathematical scenarios include exceptions and dealing with exceptions in a worksheet are technical problems in most cases. For instance, if we talk about $ax^2 + bx + c$ as a quadratic expression, and the software generates parameter a with a pseudo-random generator, we must exclude $a = 0$. (TC)
Mathematical errors	Teaching material should be free of mathematical errors. (C) In many cases mathematical errors are consequences of poor technological knowledge, see Figure 1 and Figure 2. (TC)

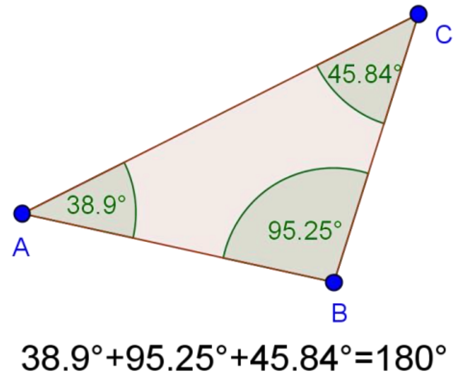

Figure 1: Screen shot from a worksheet, where rounding error occurs. Dynamic text on the left-hand side of the equation gives 179.99.

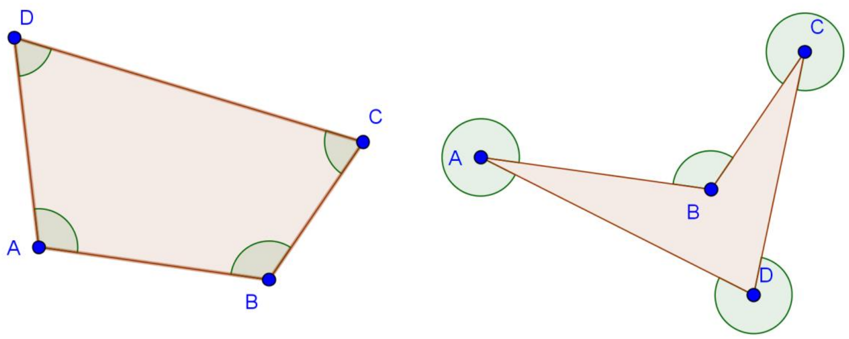

Figure 2: The figure is based on a worksheet demonstrating the inside angles in a quadrilateral with default settings (left). Moving the vertex *D* we get an erroneous drawing which leads to misconception (right).

*Pedagogical viewpoint (*Table 4*).* This aspect means the evaluation of the technical support given for the didactical aims. The student should establish the didactical aim of the worksheet or should conclude that the didactical goal is unclear. Moreover, they should investigate whether the worksheet meets the general pedagogical recommendations or not.

We distinguished two main didactical orientations of online worksheets: exercise worksheet and worksheet for knowledge support. In the first case the student should compute, group, drag something, or make a logical decision. The student is an active participant in these cases and is a subject of cognitive activity. A worksheet for knowledge support gives the student some new information, deepens or summarizes a concept, in many cases in a dynamic way. Here the pupil is the object of pedagogical activity. Examples include playing constructions, giving examples for a geometric shape, or giving solution to a problem from another teaching source.

For an example we analyse one of the above issues from pedagogical point of view (see Figure 1.) This example relates to the proving activity when using dynamic geometry software. GeoGebra supports the "confirming conjectures with measuring" strategy with a relation button directly. The decision of the machine is always associated with a warning "checked numerically". Figure 1 represents an example of this strategy, now in the form of dynamic text. In this case the numerical approximation leads to a false statement. Apart from this fault, this strategy has many shortcomings in this case. The measures of angles appear in the drawing and in the dynamic text line. The dynamic text has meaning, if (and only if) the pupil compares these numbers with the ones in the drawing. This action may lead to heavy cognitive load. Finally, the teacher knows that there is (or should be) a dynamic text on both sides of the equation. Is it enough for the student when the right hand side is a constant? The only argument is the authority of the teacher, or the student should know technical details about the software. We look at this strategy as a possible trap of dynamic geometry tools, and it should be used with caution.

Table 4: TPCK analysis of pedagogical identifiers

Identifier	Analysis (TPCK)
Support for didactical aim	An exercise worksheet contains the necessary conceptual elements i.e. hint, guiding questions, and evaluation (P), often in interactive manner and using conditional objects (T). Other elements may occur, i.e. input box, check box, usage of the pseudo random generator et cetera. (T) The content under these conceptual elements give the "Content" part.
Adherence to pedagogical principles	There are too many pedagogical principles to summarize here, and we highlight only two of them. The first is the *use examples* principle and its connection with *develop mathematical reasoning ability* intention. 'Confirming conjectures through measurement' strategy is always connected with numerical computation of the software on examples. Thus, the general didactical principles of 'reasoning and proving' widens with technical perspectives. (TP) The effort to avoid *spurious correlations*; (see [1], [2]) is the second principle here. While we give examples for a concept, it is important to avoid unnecessary limits. Limited range of parameters, unequal role of mathematically coequal elements may lead to spurious correlation. For instance, this is the case if we talk about a general quadratic expression $ax^2 + bx + c$, and restrict parameter a to positive values in our examples. (TP, TC)

The worksheet evaluation part of the course was as follows. We first discussed the evaluation guideline and illustrated with examples and then jointly evaluated a worksheet to ensure that each student has a good understanding of the viewpoints. Secondly, students evaluated one or two worksheets from the GeoGebraTube individually[1]. During the research, the authors of the present study evaluated the students' report in two phases. In Phase 1 we elaborated the peer-reviews, i.e. the researchers evaluated the same worksheets as students using the same guideline. In Phase 2 we compared the peer review and student's review. In this process, we examined whether a peer's review item appears in the student's review or not.

[1] www.geogebra.org was used in 2011.

4. The results

Our findings are based on the assessment of students' evaluation reports. We assessed 310 evaluation reports from 252 students.

In Table 5 one finds the five viewpoints, the number of peer review items and number of student review items. The 2nd line of Table 5 ("Number of student review items") refers to the number of cases in which student's and peer's evaluation of items agree. We have omitted elements from students' reviews that were not included in the peers' reviews.

Table 5: Review items in 310 evaluation reports

	Mathematical viewpoint	Pedagogical viewpoint	Layout	Dynamical elements	Text and instruction	All items
Number of peer review items (p)	772	793	942	558	756	3821
Number of student review items (s)	300	413	440	301	247	1661
Ratio (p/s)	**0.39**	**0.52**	**0.46**	**0.54**	**0.33**	**0.43**

The main finding of this study is that the peer-review cannot be eliminated from the evaluation process. Novice users analysed only less than half of the peer-review items (43%) in our research. It means that beside the possible primary role of the end user review, the peer review should not be eliminated. The low result of the mathematical viewpoint can be explained by the lack of awareness of the curriculum and teaching sources by the prospective teachers (see the first row of Table 3).

6. Conclusions

Our thesis is that the teacher's personal involvement in the evaluation-qualification process is essential in technology integration problems. In this paper we focused on the importance of qualitative analysis of online GeoGebra materials. We completed the evaluation guideline by Hohenwarter and Preiner [5] based on TPCK paradigm and tested the new evaluation framework in practice. We found that although imperfect, the novice GeoGebra user can identify values or drawbacks of an online material using this framework, but a robust peer review is necessary for quality purposes.

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